Washington University in St. Louis

Engineering

Introduction to Advanced Data Structures and Algorithms

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Analysis of Algorithms

- Why analyze algorithms?
  - » evaluate algorithm performance
  - » compare different algorithms
- Analyze what about them?
  - » running time, memory usage, solution quality
  - » worst-case and “typical” case
- Computational complexity
  - » understanding intrinsic difficulty
    - classifying problems according to difficulty
  - » algorithms provide upper bound
  - » to show problem is hard, must show that *any algorithm* to solve it requires at least a given amount of resources
    - transform problems to establish “equivalent” difficulty

Computational Problems

- Informally, a *computational problem* can be described in terms of
  - form of input provided to an algorithm for the problem
  - form of the output such an algorithm should produce
  - the relationship between input and output
- ***SORTING***
  INPUT: A list of integers $A=(a_1,\ldots,a_n)$
  OUTPUT: A list of integers $B=(b_1,\ldots,b_n)$ such that B is a permutation of A and $b_1 \leq b_2 \leq \cdots \leq b_n$
- ***MATCHING***
  INPUT: A graph $G=(V,E)$ and an integer k
  OUTPUT: A set $M \subseteq E$ such that $|M|=k$ and such that no vertex in V is incident to more than one edge of M

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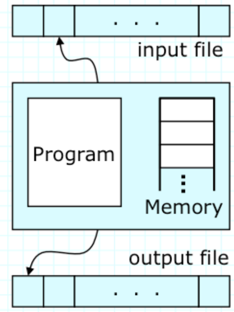
Random Access Machine



- Abstract computational model with
  - » a fixed and finite *program*
  - » an unbounded *memory*
  - » a read-only *input file*
  - » a write-only *output file*
- Each *memory register* can hold an arbitrary integer
- Each *tape cell* can hold a single symbol from a finite *alphabet* Σ
- Instruction set:
  - » $x \leftarrow y$, $x \leftarrow y \{+,-,*,\ldots\} z$
  - » goto *label*
  - » if $y \{<, =, \ldots\} z$ goto *label*
  - » $x \leftarrow$ input, output $\leftarrow y$
- Addressing modes:
  - » x may be direct or indirect reference
  - » y and z may be constants, direct or indirect references

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Asymptotic Analysis

- Focus on growth rate of running times
  - » simplifies analysis
  - » yields results that are largely independent of details of computational model
- Let f, g be functions from the non-negative integers to the positive reals
  - » say "f is $O(g)$" if there are positive constants c, n_0 such that $0 \le f(n) \le cg(n)$ for all $n > n_0$
  - » say "f is $\Omega(g)$" if there are positive constants c, n_0 such that $0 \le cg(n) \le f(n)$ for all $n > n_0$
  - » for example: $\log n$ is $O(n)$ and n^2 is $\Omega(n^2 - n)$

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Dose of Reality

- Classical analysis neglects some important factors
- Memory-latency gap
  - » in real processors, access to main memory takes ≈100 ns
    - time enough for processor to execute hundreds of instructions
  - » caches save recently-used results on-chip to avoid main memory accesses
    - relies on *locality of reference* observed in typical programs
    - programs using large linked data structures can exhibit poor locality of reference leading to poor cache performance
- Newer multi-threaded/multi-core processors require algorithms that can exploit parallelism
  - » quad core and eight core processors with multiple threads per core now common, many more coming soon

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Algorithmic Notation

- *Intervals*. An interval [$j..k$] denotes sequence $j,\ldots,k$
  [$j,k..m$] denotes the sequence $j,k, j+2(k-j),\ldots,m$
  - *example*: [1,3..7] denotes the sequence 1,3,5,7
- *Lists*. A *list* $q=[x_1,\ldots,x_n]$ is a sequence of elements; x_1 is the *head*, x_n is the *tail*. Basic list operations:
  - *Access*: If $q=[x_1,\ldots,x_n]$, $q(i) = x_i$
  - *Sublist*: $q[i..j]=[x_i,\ldots,x_j]$
  - *Concatenation*: If $r=[y_1,\ldots,y_m]$, q & $r=[x_1,\ldots,x_n,y_1,\ldots,y_m]$
- *Sets*. A *set* $s=\{x_1,\ldots,x_n\}$ is unordered collection of distinct items; basic operations are *union* ∪, *intersection* ∩ and *difference* –
- *Maps*. A *map* $f=\{[x_1,y_1],\ldots,[x_n,y_n]\}$ is set of ordered pairs, no two having same first coordinate
  - *domain*$(f)=\{x_1,\ldots,x_n\}$ and *range*$(f) = \{y_1,\ldots, y_n\}$
  - assignment $f(x):=y$ adds the pair $[x,y]$ to f

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- *Assignment*

  $x_1,\ldots,x_n := expression$

  $x_1,\ldots,x_n := exp_1 ,\ldots, exp_n$

  $x \leftrightarrow y$

- *If statement*

  if $condition_1 \Rightarrow statement\ list_1$

  | . . .

  | $condition_n \Rightarrow statement\ list_n$

  fi

- *Do statement*

  do $condition_1 \Rightarrow statement\ list_1$

  | . . .

  | $condition_n \Rightarrow statement\ list_n$

  od

- *For statement*

  for *iterator* ⇒ *statement list* **rof**

- *Subroutines*

  procedure *name*(*parameter list*); *statement list* **end**

  type **function** *name*(*parameter list*); *statement list* **end**

  predicate *name*(*param list*); *statement list* **end**

  return or

  return *expression*

Example. Binary search

```
integer function search(list s, integer x, lo, hi);
   integer mid;
   if lo > hi ⇒ return 0 fi;
   mid := ⌊(lo + hi)/2⌋;
   if s(mid) = x ⇒ return mid
     | s(mid) < x ⇒ return search(s,x,mid+1,hi)
     | s(mid) > x ⇒ return search(s,x,lo,mid-1)
   fi
end;
```

Index-Based Data Structures

- Consider list in which list items are subset of [1..*n*]
  - » such a list can be implemented as an array of *next* values
    - for item *i* not on list, let *next*[*i*]=−1 for fast membership test
    - [7,5,3,8,2]

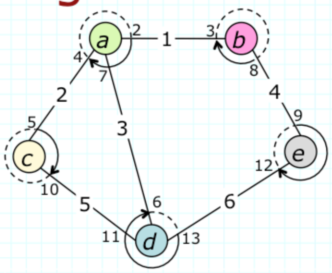
	1	2	3	4	5	6	7	8	9	10
next	−1	0	8	−1	3	−1	5	2	−1	−1

- *ListSet* defined on list items in [1..*n*]
  - » each item belongs to exactly one list (possibly singleton)
  - » implement as circular lists in shared arrays *next* and *prev*
  - » [1,6,3],[4], [2,7],[8], [5,12,10]

	1	2	3	4	5	6	7	9	8	10	11	12
next	6	7	1	4	12	3	2	11	8	5	9	10
prev	3	7	6	4	10	1	2	11	8	12	9	5

- Index values can be used as common "handle" in multiple data structures

Algorithmic Representations of Graphs



edges

	left	right
1	*a*	*b*
2	*a*	*c*
3	*d*	*a*
4	*b*	*e*
5	*c*	*d*
6	*e*	*d*

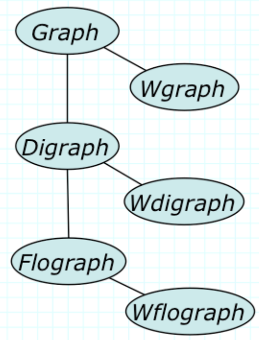
first_endpoint

a	2
b	8
c	5
d	13
e	9

adjLists

	2	3	4	5	6	7	8	9	10	11	12	13
next	7	8	2	10	13	4	3	12	5	6	9	11
prev	4	8	7	10	11	2	3	12	5	13	9	6

C++ class hierarchy for graph types



- Basic graph provides methods for traversing adjacency lists, adding edges and IO
  - » adjacency lists based on endpoints 2*e*, 2*e*+1 for edge *e*
- Variants add additional data and methods

Exercises

1. For each of the following problems, give a precise statement of the problem in the style used on page 3.

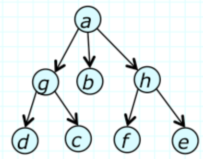
   Testing if a given string is a palindrome (reads the same way forwards and backwards).

   INPUT: Character string $s=a_1a_2 \ldots a_n$.
   OUTPUT: True if $a_i=a_{n+1-i}$ for $1 \le i \le n$, else False.

   Find a Hamiltonian cycle (a simple cycle that includes every vertex) in an undirected graph.

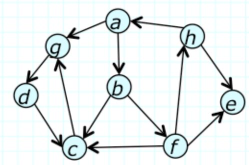
   INPUT: An undirected graph $G=(V,E)$ with n vertices.
   OUTPUT: A list of vertices $u_1,u_2, \ldots ,u_n$ where $u_i \in V$ for all i, $u_i \ne u_j$ for all $i \ne j$ and $\{u_i,u_{i+1}\} \in E$ for $1 \le i \le n$ and $\{u_n,u_1\} \in E$.

2. For the directed tree below, list the vertices in the order they would be visited by a pre-order traversal and a post-order traversal.



preorder: *a g d c b h f e*
postorder: d c g b f e h a

3. In the directed graph below, list the vertices in the order they would be visited by a depth-first search and by a breadth-first search, starting from vertex *a*. Assume that the adjacency lists are sorted by their "far" endpoints



depth-first: a b c g d f e h
breadth-first: a b g c f d e h

4. Complete the missing entries in the following data structure representing an undirected graph.

edges

	left	right
1	*a*	*b*
2	*c*	*e*
3	*e*	*d*
4	*c*	*b*
5	*d*	*c*
6	*a*	*d*

first_endpoint

a	12
b	
c	
d	7
e	

adjLists

	2	3	4	5	6	7	8	9	10	11	12	13
next	12										2	
prev												

Draw a picture of the graph.